

```
SetDirectory["C:/drorbn/AcademicPensieve/Projects/Arrow_Diagrams_and_gl(N)"];
<< "Arrow_Diagrams_and_gl(N).m"
```

se[1,i,j] stands for Aij;

se[2,i,j] stands for Bij;

se[3,i,j] stands for Dij.

Commutation relations come from <http://katlas.math.toronto.edu/drorbn/bbs/show?shot=Leung-090227-132523.jpg>

## Testing

```
bb[se1_, se2_, se3_] := Expand[b[se1, se2] /. se0_se -> b[se0, se3]];
Jac[se1_, se2_, se3_] := bb[se1, se2, se3] + bb[se2, se3, se1] + bb[se3, se1, se2]

b[se[111], se[112]]

se[112]

bb[se[111], se[112], se[112]]

0

Jac[se[111], se[114], se[214]]

0

Jac[se[343], se[221], se[113]]

0

Union[Flatten[Outer[
  (Jac[
    se[salpha[#1[[1]], #2[[1]], #2[[2]]],
    se[salpha[#1[[2]], #2[[3]], #2[[4]]],
    se[salpha[#1[[3]], #2[[5]], #2[[6]]]
  ] *
  J[
    se[salpha[#1[[1]], #2[[1]], #2[[2]]],
    se[salpha[#1[[2]], #2[[3]], #2[[4]]],
    se[salpha[#1[[3]], #2[[5]], #2[[6]]]
  ]) &,
  OrderTypes[3], OrderTypes[6], 1
]]]

{0}

{s1, s2, s3} = {se[111], se[313], se[223]}

{se[111], se[313], se[223]}

Jac[s1, s2, s3]

0

b[s1, s2]

-se[313]
```

**Diagrams [2 ar]**

```
{Diag[ar[1, 2], ar[3, 4]], Diag[ar[1, 2], ar[4, 3]], Diag[ar[2, 1], ar[3, 4]],
  Diag[ar[2, 1], ar[4, 3]], Diag[ar[1, 3], ar[2, 4]], Diag[ar[1, 3], ar[4, 2]],
  Diag[ar[2, 4], ar[3, 1]], Diag[ar[3, 1], ar[4, 2]], Diag[ar[1, 4], ar[2, 3]],
  Diag[ar[1, 4], ar[3, 2]], Diag[ar[2, 3], ar[4, 1]], Diag[ar[3, 2], ar[4, 1]]}
```

**USO[Diag[ar[1, 2], ar[3, 4]]]**

$$\begin{aligned} & \frac{1}{4} n h[1]^2 \text{RSW}[] - \frac{1}{2} n h[1]^2 \text{RSW}[111, 111] - \frac{1}{2} n^2 h[2]^2 \text{RSW}[111, 111] + 2 n^2 \text{RSW}[112, 121] - \\ & n^2 h[1]^2 \text{RSW}[112, 121] - n^2 h[2]^2 \text{RSW}[112, 121] - n^3 h[3]^2 \text{RSW}[112, 121] + n^3 \text{RSW}[113, 131] - \\ & n^3 h[2]^2 \text{RSW}[113, 131] - \frac{1}{2} n^2 h[1]^2 \text{RSW}[122, 122] - n^3 h[1]^2 \text{RSW}[123, 132] + 2 n^2 \text{RSW}[212, 312] + \\ & 2 n^3 \text{RSW}[212, 312] - n^2 h[1]^2 \text{RSW}[212, 312] - n^2 h[2]^2 \text{RSW}[212, 312] - n^3 h[3]^2 \text{RSW}[212, 312] + \\ & 2 n^3 \text{RSW}[213, 313] - n^3 h[2]^2 \text{RSW}[213, 313] + 2 n^3 \text{RSW}[223, 323] - n^3 h[1]^2 \text{RSW}[223, 323] - \\ & n^2 \text{RSW}[111, 112, 121] - n^2 \text{RSW}[111, 212, 312] + n^2 \text{RSW}[112, 121, 122] + n^3 \text{RSW}[112, 123, 131] - \\ & n^3 \text{RSW}[112, 223, 313] - n^3 \text{RSW}[113, 121, 132] + n^3 \text{RSW}[113, 223, 312] - n^3 \text{RSW}[121, 213, 323] - \\ & n^2 \text{RSW}[122, 212, 312] - n^3 \text{RSW}[123, 213, 312] + n^3 \text{RSW}[131, 212, 323] - n^3 \text{RSW}[132, 212, 313] + \\ & \frac{1}{4} n \text{RSW}[111, 111, 111, 111] + n^2 \text{RSW}[111, 111, 112, 121] + \frac{1}{2} n^2 \text{RSW}[111, 111, 122, 122] + \\ & n^3 \text{RSW}[111, 111, 123, 132] + n^2 \text{RSW}[111, 111, 212, 312] + n^3 \text{RSW}[111, 111, 223, 323] + \\ & n^2 \text{RSW}[112, 112, 121, 121] + 2 n^3 \text{RSW}[112, 113, 121, 131] + n^2 \text{RSW}[112, 121, 122, 122] + \\ & 2 n^3 \text{RSW}[112, 121, 123, 132] + n^3 \text{RSW}[112, 121, 133, 133] + 2 n^4 \text{RSW}[112, 121, 134, 143] + \\ & 2 n^2 \text{RSW}[112, 121, 212, 312] + 2 n^3 \text{RSW}[112, 121, 213, 313] + 2 n^3 \text{RSW}[112, 121, 223, 323] + \\ & 2 n^4 \text{RSW}[112, 121, 234, 334] + n^3 \text{RSW}[113, 122, 122, 131] + 2 n^3 \text{RSW}[113, 123, 131, 132] + \\ & 2 n^4 \text{RSW}[113, 124, 131, 142] + 2 n^3 \text{RSW}[113, 131, 212, 312] + 2 n^3 \text{RSW}[113, 131, 223, 323] + \\ & 2 n^4 \text{RSW}[113, 131, 224, 324] + 2 n^4 \text{RSW}[114, 123, 132, 141] + 2 n^4 \text{RSW}[114, 141, 223, 323] + \\ & n^2 \text{RSW}[122, 122, 212, 312] + n^3 \text{RSW}[122, 122, 213, 313] + 2 n^3 \text{RSW}[123, 132, 212, 312] + \\ & 2 n^3 \text{RSW}[123, 132, 213, 313] + 2 n^4 \text{RSW}[123, 132, 214, 314] + 2 n^4 \text{RSW}[124, 142, 213, 313] + \\ & n^3 \text{RSW}[133, 133, 212, 312] + 2 n^4 \text{RSW}[134, 143, 212, 312] + n^2 \text{RSW}[212, 212, 312, 312] + \\ & 2 n^3 \text{RSW}[212, 213, 312, 313] + 2 n^3 \text{RSW}[212, 223, 312, 323] + 2 n^4 \text{RSW}[212, 234, 312, 334] + \\ & 2 n^3 \text{RSW}[213, 223, 313, 323] + 2 n^4 \text{RSW}[213, 224, 313, 324] + 2 n^4 \text{RSW}[214, 223, 314, 323] \end{aligned}$$
**USO[Take[R/@Diagrams[1 ar + R6T], 2]]**

```
{0, 0}
```

**l = Length[rels = R/@Diagrams[1 ar + R6T]]**

```
120
```

**USO[List @@ rels[[1]]]**

A very large output was generated. Here is a sample of it:

$$\left\{ -\frac{1}{8} n h[1]^2 \text{RSW}[] + \frac{3}{8} n h[1]^2 \text{RSW}[111, 111] + \frac{3}{8} n^2 h[2]^2 \text{RSW}[111, 111] + \ll 2842 \gg + \right.$$

$$6 n^6 \text{RSW}[215, 224, 236, 315, 324, 336] + 6 n^5 \text{RSW}[215, 225, 234, 315, 325, 334] +$$

$$6 n^6 \text{RSW}[215, 226, 234, 315, 326, 334] + 6 n^6 \text{RSW}[216, 223, 245, 316, 323, 345] +$$

$$6 n^6 \text{RSW}[216, 224, 235, 316, 324, 335] + 6 n^6 \text{RSW}[216, 225, 234, 316, 325, 334],$$

$$\left. -\frac{1}{8} n h[1]^2 \text{RSW}[] + \ll 2829 \gg + 6 n^6 \text{RSW}[216, 225, 234, 316, 325, 334], \ll 1 \gg \right\}$$
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```

Total[USO[List @@ rels[[1]]]]

0

l = Length[rels = R /@ Diagrams[1 ar + R6T]];
Table[
  Print[{i, rels[[i]], Timing[uso = USO[rels[[i]]]}];
  uso,
  {i, l}
]

Computing OrderTypes[6, {{1, 2}, {3, 4}, {5, 6}}]
Computing OrderTypes[6, {{3, 4}, {5, 6}}]
Computing OrderTypes[6, {{5, 6}}]
Computing OrderTypes[5, {{1, 2}, {3, 4}}]
Computing OrderTypes[5, {{3, 4}}]
Computing OrderTypes[3, {{1, 2}}]
Computing OrderTypes[4, {{1, 2}}]
Computing OrderTypes[5, {{1, 2}, {4, 5}}]
Computing OrderTypes[5, {{4, 5}}]
Computing OrderTypes[3, {{2, 3}}]
Computing OrderTypes[4, {{3, 4}}]
Computing OrderTypes[5, {{2, 3}, {4, 5}}]
{11 670, 2077}
{11 670, 2060}
{11 670, 2075}
{1, Diag[ar[1, 2], ar[3, 4], ar[5, 6]] +
  Diag[ar[1, 2], ar[3, 5], ar[4, 6]] - 2 Diag[ar[1, 2], ar[3, 6], ar[4, 5]], {16.848, 0}}
{11 670, 3310}
{11 670, 3311}
{2, Diag[ar[1, 2], ar[3, 4], ar[6, 5]] - Diag[ar[1, 2], ar[3, 5], ar[4, 6]] +
  Diag[ar[1, 2], ar[3, 6], ar[4, 5]] - Diag[ar[1, 2], ar[3, 6], ar[5, 4]], {19.157, 0}}
{11 670, 3310}
{3, -Diag[ar[1, 2], ar[3, 5], ar[4, 6]] + Diag[ar[1, 2], ar[3, 6], ar[4, 5]] -
  Diag[ar[1, 2], ar[3, 6], ar[5, 4]] + Diag[ar[1, 2], ar[4, 3], ar[5, 6]], {9.485, 0}}
{11 670, 3358}
{11 670, 3575}
{11 670, 3587}
{4, -Diag[ar[1, 2], ar[4, 3], ar[5, 6]] + Diag[ar[1, 2], ar[4, 5], ar[6, 3]] +
  Diag[ar[1, 2], ar[5, 3], ar[6, 4]] - Diag[ar[1, 2], ar[5, 4], ar[6, 3]], {44.99, 0}}
{5, -Diag[ar[1, 2], ar[3, 4], ar[6, 5]] + Diag[ar[1, 2], ar[4, 5], ar[6, 3]] +
  Diag[ar[1, 2], ar[5, 3], ar[6, 4]] - Diag[ar[1, 2], ar[5, 4], ar[6, 3]], {0.094, 0}}

```

```
{11 670, 3542}
{6, -Diag[ar[1, 2], ar[4, 3], ar[6, 5]] -
  Diag[ar[1, 2], ar[5, 3], ar[6, 4]] + 2 Diag[ar[1, 2], ar[5, 4], ar[6, 3]], {16.302, 0}}
{11 670, 3310}
{11 670, 3287}
{11 670, 3288}
{7, Diag[ar[2, 1], ar[3, 4], ar[5, 6]] +
  Diag[ar[2, 1], ar[3, 5], ar[4, 6]] - 2 Diag[ar[2, 1], ar[3, 6], ar[4, 5]], {29.531, 0}}
{11 670, 3542}
```

\$Aborted

**USO[Diag[ar[1, 2], ar[3, 4], ar[5, 6]]]**

A very large output was generated. Here is a sample of it:

$$-\frac{1}{8} n h[1]^2 \text{RSW}[] + \frac{3}{8} n h[1]^2 \text{RSW}[111, 111] + \frac{3}{8} n^2 h[2]^2 \text{RSW}[111, 111] + \ll 2842 \gg +$$

$$6 n^6 \text{RSW}[215, 224, 236, 315, 324, 336] + 6 n^5 \text{RSW}[215, 225, 234, 315, 325, 334] +$$

$$6 n^6 \text{RSW}[215, 226, 234, 315, 326, 334] + 6 n^6 \text{RSW}[216, 223, 245, 316, 323, 345] +$$

$$6 n^6 \text{RSW}[216, 224, 235, 316, 324, 335] + 6 n^6 \text{RSW}[216, 225, 234, 316, 325, 334]$$

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**diag = BasisAArrow[4][[101]]**

Diag[ar[1, 8], ar[2, 7], ar[5, 3], ar[6, 4]]

**USO[diag]**

```
{1 390 450, 987 535}
```

\$Aborted

**diag1 = BasisAArrow[4][[1]]**

Diag[ar[5, 1], ar[6, 4], ar[7, 3], ar[8, 2]]

**USO[diag1]**

```
{1 390 450, 1 143 509}
```

\$Aborted